Errata of "Digital and Discrete Geometry"

1. On Page 174, Equation (10.4) should be changed by adding $\eta(t)$ in the original equation.

10.2.5 Curvatures

The curvature measures the degree of the curve. It can be viewed as the magnitude of the acceleration of the curve with a special local normalization.

$$\kappa(t) = \|s''(t)\|\eta(t). \tag{10.4}$$

$s''(t)$ is the second derivative with respect to t; $\eta(t) = \frac{|\sin(\theta)|}{\|s'(t)\|^2}$, where θ is the angle between s' and s''. When s is a straight line, $s''(t) = 0$, then there is no curvature. When s is a circle, then θ is 90 degrees. $\kappa(t) = \|s''(t)/|s'(t)|^2\|$. This format is similar to that of the Riemann curve in Fig. 13.8 of Chap. 13.

Mathematically, the definition of curvature is the derivative of the unit vector of tangent line $s'(t)$ with respect to the curve length. Let $T = \frac{s'}{|s'|}$, $\kappa(t) = \|\frac{dT(t)}{d\rho}\|$, ρ be the arc length of s, and T indicate the angle. Therefore, curvatures can also be defined as the magnitude of the rate of change in the angle to the rate of change in the length $\frac{d\phi}{d\rho}$. The equivalence between $\|s''(t)\|\eta(t)$ and $\frac{d\phi}{d\rho}$ can be derived.

The radius of the curvature $R(t)$ is the reciprocal of the radius of curvature:

2. On Page 232, the curvature referring to (10.4) should be changed accordingly.

3. On Page 159, **Line 24**, $H_0=Z$. This is because there is only one connected component. **On Line 25**, $H_2(X)=0$ since one 2-cell cannot make a cycle made by 2-cells.

   To understand why H_0 is Z^k where k is the number of connected components in X, we can see ∂_0 send every point to 0. So $Ker(\partial_0)$ contains every 0-cell as generator. We now consider $Im(\partial_1)$. ∂_1 sends every 1-cell to its boundary. That are two 0-cells. Even though that it is not intuitively true, but two 0-cells of a 1-cell is a 0-cycle. This is the only special case. For an n-disk, its boundary is a (n-1)-cycle. When $n>1$, it is intuitively correct as well. We need to remove the group generated by the 0-cycle (two 0-cells) from $Ker(\partial_0)$. That is just one Z. Then we get a reduced $Ker(\partial_0)$. For every 1-cell in *X*, we will withdraw a *Z* from $Ker(\partial_0)$. So we will end up with $H_0=Z$ if there is only one connected component. We can also see that H_0 is Z^k where *k* is the number of connected components in *X*.

For example, a triangle (not filled) has three edges (1-cells), a,b,c with vertices (0-cells) A,B,C. a→<A,B> (or *g=A-B* that is an element in group C_0), b→B-C, and C→C-A. So *C-A = -((A-B)+(B-C))* in the group. We only use two generators (A-B) and (B-C). So $Im((\partial_1)=Z^2$. $Ker(\partial_0)=Z^3$ since every A,B,C is generator. So, $H_0=Z$. For the same reason, if a component has *t* vertices, we only need *(t-1)* edges to link them (a spanning tree), the rest of edges is the combination of the subset of 0-cells in the tree. $H_0=Z$ for this component.

The homology group is *cycles modulo boundaries* (the boundary of a *(i+1)*-object). Boundary is always a cycle or a set of cycles, but cycle may not be a boundary of a ``solid.'' Homology is to count the cycle that is not a boundary. And the number of minimal cycles, not the one can be generated in terms of a group.